NOTATION

r, z, cylindrical coordinates; u_r, u_z, projections of the vapor velocity vector onto the coordinate axes; T, T_ℓ, temperatures in the vapor layer and in the liquid spheroid; μ, a, coefficients of vapor viscosity and thermal conductivity; P, pressure within the vapor layer; V, velocity of the vapor at the boundary of phase separation; λ and λ_ℓ, coefficients of vapor and liquid thermal conductivity; T_s, T_w, T_c, saturation temperatures for the liquid, the heated surface, and the ambient medium; P_c, pressure in the ambient medium; ρ, ρ_ℓ, density of the vapor and of the liquid; L, specific heat of vaporization; α_1, α_2, heat-transfer coefficients at the upper and side surfaces of the spheroid; H, R, thickness and radius of the spheroid; h, thickness of the vapor layer; g, gravitational acceleration; J_0, J_1, Bessel functions of the first kind, of zeroth and first order; h_*, ΔT, characteristic values of the vapor-layer thickness and the temperature difference; Δ, roughness magnitude of the heated surface; T_L, T_L', Leidenfrost temperature for the cases in which consideration is given to and not given to the exchange of heat between the liquid spheroid and the ambient medium. Criteria: $Bi_1 = \alpha_1 R/\lambda_\ell$; $Bi_2 = \alpha_2 R/\lambda_\ell$.

LITERATURE CITED

1. S. S. Kutateladze, Heat Transfer in Condensation and Boiling [in Russian], Moscow—Leningrad (1949).
2. V. M. Borishanskii, Questions of Heat Exchange with a Change in the Aggregate State of Matter [in Russian], Moscow—Leningrad (1953), pp. 118-155.
3. L. H. J. Wachters, H. Bonne, and H. J. Van Nouhuis, Chem. Eng. Sci., 21, 923-936 (1966).
4. Yu. A. Buevich and V. N. Mankevich, Inzh.-Fiz. Zh., 44, No. 6, 949-953 (1983).
5. M. A. Goldshtik, V. M. Khanin, and V. G. Ligai, J. Fluid Mech., 166, 1-20 (1986).
6. N. A. Gezekhus, J. Russian Chem. Soc. and Phys. Soc., Physics Section, 8, No. 6, 311-343; No. 7, 356-399 (1876).
7. A. V. Lykov, The Theory of Heat Conduction [in Russian], Moscow (1967).

THE SPREADING OF A MICROSTRUCTURAL FLUID OVER A SOLID SURFACE

N. P. Migun and A. C. Eringen

UDC 532.5.013

The kinetics involved in the spreading of a drop of microstructural fluid over a horizontal solid surface is investigated theoretically. A method is proposed for the measurement of material constants of the fluid, characterizing its micropolarity.

1. Specialists in the field of hydrodynamics and physical chemistry have recently paid particular attention to problems of fluid spreading and displacement of the contact line between fluid 1, fluid 2, and a solid surface [1-9]. Considerable progress has been achieved at this time in this area, but at the same time all of the attempts theoretically to analyze the problems of spreading encounter two fundamental difficulties.

The first difficulty involves the shifting of the contact line, since the Navier—Stokes equations for the boundary conditions of adhesion lead to an impermissible singularity in the force on this line [1, 2]. There exists a means of eliminating this singularity of force by means of utilizing the condition of slippage or shear in the region of the contact line, e.g., the Maxwell condition at which the magnitude of the shear is proportional to the local velocity gradient [3, 4].

Institute of Applied Physics, Academy of Sciences of the Belorussian SSR, Minsk. Princeton University, USA. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 2, pp. 253-261, February, 1989. Original article submitted July 30, 1987.

0022-0841/89/5602-0173$12.50

The second difficulty encountered in theoretical analysis of drop spreading is associated with the problem of the contact wetting angle. It is obvious that the contact angle depends on time. For example, a fairly common approach is one in which this relationship is associated with the angle formed by the tangent to the drop surface not at the contact line, but at some distance from it [4]. However, the real contact wetting angle on the contact line itself is assumed to be constant.

There now exist a number of solutions for the problem of drop spreading over a horizontal surface, but the procedure involved in obtaining the majority of these solutions is rather complex. For example, in utilizing the shear condition we introduce the parameter of characteristic length which is a measure of the extent of the region near the contact line, within which this shear occurs. The problem is then formulated for two or three regions and by means of the "matching" procedure for asymptotic expansions we derive an equation for the velocity of drop spreading as a function of the wetted-surface radius. The equation is solved numerically.

As far as we know, the first quantitative description of the kinetics of quasisteady spreading of a drop of Newtonian fluid over a horizontal solid surface, based on a comparison of the free surface energy and the work required to overcome viscous friction, was presented in the article [7]. The problem of the spreading of a drop of Newtonian fluid in the case of very good wetting (when the contact angle $\theta \to 0$) for a quasisteady process was solved through the use of a series of assumptions relative to the Navier–Stokes and continuity equations by the authors of [8]. A relationship was derived between time and the radius of the circle formed by a wetted drop of Newtonian fluid over a solid surface, and this relationship proved to be in good agreement with experimental data for a number of fluids. The approach to the solution of the spreading problem as described in [8] becomes attractive because of its simplicity. With this approach it is possible to avoid the two-dimensionality of the problem and the difficulties associated with the formulation of the boundary conditions. As regards the problem of the contact angle, it is appropriate to assume (as is done, for example, in [4]) that the magnitude of the contact wetting angle in the vicinity of the contact line is independent of time. Let us note that it is precisely this contact angle that characterizes the dynamic spreading coefficient [10].

Below we examine the spreading of a microstructure fluid over a horizontal solid surface in the case of wetting, very nearly total, as $\theta \to 0$. Fluids whose volume microelements may also exhibit, in addition to translational velocity $\mathbf{v}$, intrinsic angular velocity $\boldsymbol{\nu}$, differing from a vortex $\mathbf{w} = (1/2) \times \operatorname{rot} \mathbf{v}$, are referred to as microstructural fluids. These include, for example, magnetorheological, electrorheological, and certain other types of suspensions and liquid crystals, blood, associated fluids (for example, water). The condition under which the field of intrinsic rotations $\boldsymbol{\nu} \neq \mathbf{w}$ arises is comparability of characteristic flow dimension (for example, the thickness of the layer or of the capillary radius) and the average dimension of the fluid's microelement.

In this connection it should be noted that when a drop of a microstructural fluid spreads out over a horizontal solid surface in the case of $\cos\theta \simeq 1$ the fluid spreads out in a very thin layer whose thickness rapidly diminishes with time. This means that, beginning from a particular drop radius, the intrinsic rotations of the fluid's microelements begin, with the passage of time, to exert an increasing influence on the nature of the spreading.

2. Formulation and Solution of the Problem. Let us examine the spreading of a drop of microstructural fluid of mass m over a horizontal solid surface with nearly total wetting as $\cos\theta \to 1$. For a description of the process we will employ the theory of micropolar fluids [11] within whose scope allowance is made not only for translational but intrinsic rotational velocities of the microelements of the medium's volume. The differential equations for the velocity $\mathbf{v}$ and microrotation $\boldsymbol{\nu}$ of the micropolar fluid are written as follows in general form [11]:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0,$$

$$(\lambda + 2\mu + \varkappa)\nabla\nabla\cdot\mathbf{v} - (\mu + \varkappa)\nabla\times\nabla\times\mathbf{v} + \varkappa\nabla\times\boldsymbol{\nu} - \nabla\pi^\circ + \rho\mathbf{f} =$$

$$= \rho\left[\frac{\partial\mathbf{v}}{\partial t} - \mathbf{v}\times(\nabla\times\mathbf{v}) + \frac{1}{2}\nabla(\mathbf{v}^2)\right],$$

$$(\alpha_v + \beta_v + \gamma)\nabla\nabla\cdot\boldsymbol{\nu} - \gamma\nabla\times\nabla\times\boldsymbol{\nu} + \varkappa\nabla\times\mathbf{v} - 2\varkappa\boldsymbol{\nu} + \rho\mathbf{l} = \rho j\dot{\boldsymbol{\nu}}. \tag{1}$$

We will make the following assumptions: 1) we will examine the quasisteady process of spreading in which we can neglect the local derivative of velocity with respect to time in comparison with the viscous term; 2) the inertial effects are negligibly small; 3) the fluid is incompressible; 4) we will neglect the evaporation of the fluid layer during the time of the spreading, i.e., we will assume $m \simeq \text{const}$.

Let us write the system of equations (1) for the enumerated assumptions in a cylindrical system of coordinates whose z axis is perpendicular to the solid surface and coincides with the axis of symmetry for the drop:

$$
\begin{gathered}
(\mu+\varkappa)\frac{d^2v_r}{dz^2}-\varkappa\frac{d\nu_\varphi}{dz}-\frac{dp}{dr}+\rho f_r=0,\\
\gamma\frac{d^2\nu_\varphi}{dz^2}+\varkappa\frac{dv_r}{dz}-2\varkappa\nu_\varphi=0,
\end{gathered}
\tag{2}
$$

where v_r and ν_φ are the nonzero components of the vectors $\mathbf{v}$ and $\boldsymbol{\nu}$; dp/dr is the pressure gradient in the fluid layer. We will regard the spreading as a consequence solely of a reduction in the free surface energy. In accordance with the approach described in [8], the external force f_r acting on the system can be expressed as

$$
f_r=-\frac{1}{m}\,\frac{\partial(\Delta W)}{\partial r},
$$

where ΔW is the change in the free surface energy of the solid wetted by a fluid layer of mass m, and this change can be expressed in terms of the specific free surface energies at the boundary between the solid and the ambient medium σ_{sg} and with the fluid σ_{sf} as

$$
\Delta W=K(\sigma_{s\ell}-\sigma_{sg})\pi r^2,
$$

where K is the surface roughness coefficient [12]. Hence

$$
f_r=\frac{2\pi rK(\sigma_{sg}-\sigma_{s\ell})}{m}. \tag{3}
$$

The pressure gradient in this case is defined in terms of the surface tension σ_{fg} which can be presented as the force acting per unit length of the contact line:

$$
\frac{\partial p}{\partial r}=\frac{2\pi r\sigma_{\ell g}}{m/\rho}. \tag{4}
$$

Considering the fact that $K(\sigma_{sg}-\sigma_{sf})-\sigma_{fg}=S$, where S is the spreading coefficient [12], and having substituted (3) and (4) into (2), we obtain

$$
\begin{gathered}
(\mu+\varkappa)\frac{d^2v_r}{dz^2}-\varkappa\frac{d\nu_\varphi}{dz}+\frac{2\pi r\rho S}{m}=0,\\
\gamma\frac{d^2\nu_\varphi}{dz^2}+\varkappa\frac{dv_r}{dz}-2\varkappa\nu_\varphi=0.
\end{gathered}
\tag{5}
$$

This system of equations describes the spreading of a drop of micropolar fluid over a horizontal solid surface.

The boundary conditions for velocity and microrotation can be formulated in the following manner. On a nonmoving solid surface we will assume the condition of adhesion for $\mathbf{v}$ and a boundary condition of the general type for $\boldsymbol{\nu}$ [13]:

$$
\boldsymbol{\nu}\big|_b=\frac{\alpha}{2}(\nabla\times\mathbf{v})\big|_b.
$$

Consequently:

$$
v_r\big|_{z=0}=0,\quad \nu_\varphi\big|_{z=0}=\frac{\alpha}{2}\left(\frac{dv_r}{dz}\right)\Big|_{z=0}. \tag{6}
$$

The dynamic boundary conditions that must be satisfied at the free surface include absence of stresses $\mathbf{t}$ and couple stresses $\mathbf{m}$: $\mathbf{t}=\mathbf{m}=0$ on the free surface.

Since for a micropolar fluid [11] $t_{k\ell} = (-\pi^\circ + \lambda v_{r,r})\delta_{k\ell} + \mu(v_{k,\ell} + v_{\ell,k}) + \varkappa(v_{\ell,k} - e_{k\ell r}\nu_r)$, $m_{k\ell} = \alpha_v \nu_{r,r}\delta_{k\ell} + \beta_v \nu_{k,\ell} + \gamma\nu_{\ell,k}$, we can write the following boundary conditions on the free surface:

$$(\mu+\varkappa)\left(\frac{dv_r}{dz}\right)\Big|_{z=h} - \varkappa\nu_\varphi(h) = 0, \quad \frac{d\nu_\varphi}{dz}\Big|_{z=h} = 0. \tag{7}$$

Solution of the system of differential equations (5) for boundary conditions (6) and (7) leads to the following expressions for the nonzero components of velocity and microrotation:

$$\begin{aligned} v_r^{\text{mp}} &= \frac{Rh}{\mu^N \tilde{k}}\left[\left(b\,\mathrm{sh}\,\tilde{k}h + \frac{\beta}{2\tilde{k}h}\right)\frac{\mathrm{ch}\,\tilde{k}z - 1}{\mathrm{ch}\tilde{k}h} - \right. \\ &\qquad \left. - b\,\mathrm{sh}\,\tilde{k}z + \tilde{k}z - \frac{\tilde{k}z^2}{2h}\right], \\ \nu_\varphi &= \frac{Rh}{\mu^N}\left[\left(\frac{b\,\mathrm{sh}\,\tilde{k}h}{\beta} + \frac{1}{2\tilde{k}h}\right)\frac{\mathrm{sh}\,\tilde{k}z}{\mathrm{ch}\,\tilde{k}h} - \frac{b}{\beta}\,\mathrm{ch}\,\tilde{k}z - \frac{z}{2h} + \frac{1}{2}\right], \end{aligned} \tag{8}$$

where

$$b = \frac{\varkappa(1-\alpha)}{2\mu^N + \varkappa(1-\alpha)}, \quad \tilde{k} = \left(\frac{2\mu^N\beta}{\gamma}\right)^{1/2}, \quad \beta = \frac{\varkappa}{\mu+\varkappa}, \quad R = \frac{2\pi r\rho S}{m}. \tag{9}$$

Since in the case of a moving fluid front z = h, using (8) and (9), as well as the condition of mass constancy $m = \pi r^2\rho h$, it is easy to derive a formula for the spreading rate

$$v_r^{\text{mp}} = \frac{dr}{dt} = \frac{a}{r^3}\left\{1 + \frac{\beta[\mathrm{ch}(p/r^2) - 1] - 2(p/r^2)\,b\,\mathrm{sh}(p/r^2)}{(p/r^2)^2\,\mathrm{ch}(p/r^2)}\right\}, \tag{10}$$

where $p = m\tilde{k}/\pi\rho$, $a = mS/(\mu^N\pi\rho)$.

When $\varkappa = 0$, formula (10) changes into an expression for the velocity of a Newtonian fluid [8]

$$v_r^N = dr/dt = a/r^3. \tag{11}$$

By integrating this equation we can calculate the time t^N needed for the front of the Newtonian fluid to cover the distance from r_1 to r_2:

$$t^N = (r_2^4 - r_1^4)/4a. \tag{12}$$

An analogous formula for a micropolar fluid follows from Eq. (10):

$$t^{\text{mp}} = t^N \frac{4p^2}{r_2^4 - r_1^4}\int_{r_1}^{r_2} \frac{\xi\,\mathrm{ch}\,\xi\,d\xi}{\xi\,\mathrm{ch}\,\xi + \beta(\mathrm{ch}\,\xi - 1)/\xi - 2b\,\mathrm{sh}\,\xi}, \tag{13}$$

where $\xi = p/r^2$.

The ratios v_r^{mp}/v_r^N and t^{mp}/t^N characterize the difference in the values of velocities and times calculated both with and without consideration of the fluid's micropolarity.

<u>3. The Method of Determining the Material Constants of a Micropolar Fluid and the Parameters of the Boundary Conditions.</u> Using formulas (10) and (13), we can quantitatively describe the kinetics for the spreading of a drop of microstructural fluid over a horizontal solid surface. However, for this we must know the magnitudes of the parameters $\tilde{k}$, $\varkappa$, and α. The following method is proposed for the determination of these quantities.

Let us assume that we have used slow-motion photography to measure the spreading velocities v_{r_1} and v_{r_2}, corresponding to two values of the wetted-surface radius r_1 and r_2. Using formula (10), we can express v_{r_1} and v_{r_2} in terms of $\tilde{k}$, $\varkappa$, and α for both radii. A method has been developed in [14] to determine the parameters $\tilde{k}$ and $\varepsilon = \varkappa(1 - \alpha)$. This method was applied, for example, to calculate the microstructural parameters of certain tracer fluids used in capillary defectoscopy [14]. Consequently, we can find the quantity p which is contained in the expressions for v_{r_1} and v_{r_2}. The quantity a which contains the spreading coefficient is determined experimentally: we will calculate this quantity by using the experimental data for the spreading velocity at the beginning of the process when $\xi \gtrsim 10$ and when formula (11) is valid. Thus, the expressions for v_{r_1} and v_{r_2} represent a system of algebraic equations for two unknowns: β and b, which when transformed into an equation for β, yields:

$$\xi_2 \operatorname{sh} \xi_2 [a\xi_1^2 \operatorname{ch} \xi_1 + a\beta (\operatorname{ch} \xi_1 - 1) - Q_1] = \xi_1 \operatorname{sh} \xi_1 [a\xi_2^2 \operatorname{ch} \xi_2 + a\beta (\operatorname{ch} \xi_2 - 1) - Q_2], \tag{14}$$

where

$$Q_1 = v_{r_1} \xi_1^2 \operatorname{ch} \xi_1 r_1^3, \quad Q_2 = v_{r_2} \xi_2^2 \operatorname{ch} \xi_2 r_2^3, \quad \xi_1 = p/r_1^2, \quad \xi_2 = p/r_2^2.$$

From (14) we obtain the expression

$$\beta = \frac{1}{a} \; \frac{\xi_1 \operatorname{sh} \xi_1 (a\xi_2^2 \operatorname{ch} \xi_2 - Q_2) - (a\xi_1^2 \operatorname{ch} \xi_1 - Q_1) \xi_2 \operatorname{sh} \xi_2}{\xi_2 \operatorname{sh} \xi_2 (\operatorname{ch} \xi_1 - 1) - (\operatorname{ch} \xi_2 - 1) \xi_1 \operatorname{sh} \xi_1}. \tag{15}$$

Having determined β, with the aid of (10) we find the second parameter:

$$b = \frac{a\xi_1^2 \operatorname{ch} \xi_1 + a\beta (\operatorname{ch} \xi_1 - 1) - Q_1}{2\xi_1 \operatorname{sh} \xi_1}. \tag{16}$$

Now, bearing in mind that $2\mu^N = 2\mu + \varkappa$, and using (15), we obtain the formulas for the material constants $\varkappa$ and μ:

$$\varkappa = \frac{2\mu^N \beta}{2 - \beta}, \quad \mu = \frac{2\mu^N (1 - \beta)}{2 - \beta}. \tag{17}$$

Since

$$b = \frac{\varkappa (1 - \alpha)}{2\mu^N + \varkappa (1 - \alpha)},$$

it is not difficult to write an expression for the calculation of the boundary-condition parameter α in which the quantities b, $\varkappa$, and μ are found from formulas (16) and (17):

$$\alpha = \frac{2 (\mu + \varkappa) b - \varkappa}{\varkappa (b - 1)}. \tag{18}$$

4. The Effect of Fluid Micropolarity on the Kinetics of Fluid Spreading Over a Solid Surface. The effect of the intrinsic rotations of the microelements in the volume of a microstructural fluid (or its micropolarity) on the velocity of drop spreading over a solid surface can be described quantitatively by means of three dimensionless microstructural parameters: β, b, and ξ. The latter can be expressed in terms of the characteristic flow dimension, i.e., in this particular case, in terms of the thickness h of the spreading drop. If we take into consideration that $\xi = p/r^2$, while the mass of the drop $m = \pi r^2 \rho h$, we can write ξ as follows:

$$\xi = \tilde{k} h. \tag{19}$$

Consequently, the parameter ξ is characterized both by the physical properties of the material and by the characteristic dimension of the flow. It is obvious that the greater the average dimension of the fluid's microelements and the smaller the thickness of the spreading layer, the more markedly will the micropolarity affect the kinetics of the process. The parameter β is defined exclusively by the material constants of the fluid, whereas the parameter b is determined both from the material constants and the nature of the boundary conditions for the microrotations.

Numerical analysis of formulas (10) and (13) demonstrates that when $\xi \gtrsim 10$ even in the case of $\varkappa \gg \mu^N$, i.e., when the rotational viscosity is very much greater than the shear viscosity, the difference between v_r^{mp} and v_r^N as well as t^{mp} and t^N does not exceed 2-3%. Consequently, if we know the quantity $\tilde{k}$ of the fluid, we can estimate the magnitude of the layer thickness h_m, below which the micropolarity of the fluid influences the kinetics of fluid spreading.

The greater the polarity of the associated fluid, the smaller the parameter $\tilde{k}$, and in the case of water, a more polar fluid, $\tilde{k} = 7\cdot10^7$ m^{-1} [14]. Consequently, when $h > h_m = 1.5\cdot10^{-7}$ m the micropolarity of the associated fluids exerts virtually no effect on the velocity of spreading. In actual practice, such a layer thickness as a consequence of its evaporation during the spreading process even in the case of nearly total wetting is rarely achieved. Hence follows the conclusion that there is no merit in taking into account the micropolarity of associated fluids in the theoretical description of their spreading over a solid surface. Physically this is explained by the smallness of the dimensions d of the microelements exhibiting intrinsic rotations. For associated fluids the dimensions of such microelements (associates) are $d \simeq (1\text{-}3)\cdot10^{-9}$ m. However, numerous suspensions contain particles whose dimensions are larger by several orders of magnitude.

Let us assume that the quantity h_m/d for various fluids is constant. Since for water $h_m/d \simeq 70$, from the condition of negligibility of the micropolarity of the fluid for spreading in a layer with $h > h_m$, where $h_m \simeq 10/\tilde{k}$, there follows the relationship between the magnitudes of the microstructural parameter $\tilde{k}$ and the average dimension d of the suspension particles:

$$\tilde{k} \simeq (7d)^{-1}. \tag{20}$$

For example, let us examine suspensions with average particle dimensions of $d_1 = 3\cdot10^{-6}$ m and $d_2 = 2\cdot10^{-5}$ m. It follows from (20) that for the first suspension $\tilde{k}_1 = 5\cdot10^4$ m^{-1}, while for the second suspension $\tilde{k}_2 = 7\cdot10^3$ m^{-1}. It is also easy to demonstrate that $h_{m_1} = 2\cdot10^{-4}$ m and $h_{m_2} = 1.4\cdot10^{-3}$ m. This indicates, for example, that for a suspension with particles of diameter $d_2 = 2\cdot10^{-5}$ m the velocity of spreading diminishes in comparison with that calculated for a Newtonian fluid even at a layer thickness of $h < 1.4\cdot10^{-3}$ m.

Let a drop of a suspension applied to a horizontal solid surface have a diameter $2\cdot10^{-3}$ m and a density 10^3 kg$\cdot$m^{-3}. Then $m \simeq 3.35\cdot10^{-5}$ kg, and the parameter p for both of the suspensions under consideration assumes values of $p_1 = 5.3\cdot10^{-4}$ m^2 and $p_2 = 7.5\cdot10^{-5}$ m^2.

The layers of the drops from both of the suspensions with a diameter of $2\cdot10^{-3}$ m at a distance $r > 5\cdot10^{-3}$ m from the center of the wetted circle exhibit a thickness $h < 5\cdot10^{-4}$ m. This means that for more highly dispersed suspensions even at such a distance from the center of the circle the influence of the micropolarity of the medium is significant insofar as it pertains to the spreading velocity and it must be taken into consideration in the corresponding calculations.

Let us use formulas (10), (11), and (13) to calculate the ratios v_r^{mp}/v_r^N and t^{mp}/t^N for the micropolar fluids with various values of the material constants.

For small values of the parameter ξ formulas (10) and (13) are significantly simplified; for the case in which $\xi \to 0$

$$\frac{v_r^{\mathrm{mp}}}{v_r^{\mathrm{N}}} \to 1 + \frac{\beta}{2} - 2b, \qquad \frac{t^{\mathrm{mp}}}{t^{\mathrm{N}}} \to \left(1 + \frac{\beta}{2} - 2b\right)^{-1}. \tag{21}$$

It is obvious that not every combination of the quantities $\tilde{k}$, $\varkappa$, and α, in terms of which the parameters p and b contained in formulas (10) and (13) are expressed, will satisfy physical reality. In selecting the physically permissible combinations we will proceed primarily from the fact that the calculation of the spreading velocity with consideration of additional (rotational) degrees of freedom must lead to values of the spreading velocities that are smaller in comparison with those found from formula (11) for the Newtonian fluid. The spreading of the Newtonian fluid is characterized by a distribution of energy (in the case under consideration it is determined by a reduction in the free surface energy) only over the degrees of freedom for the translational motion of the microelements of the medium's volume and this energy is expended on overcoming the shear friction. In the case of a microstructural fluid a portion of the energy is expended also on overcoming rotational interactions, i.e., the overall reduction in free surface energy is distributed over the degrees of freedom not only of translational but also rotational motion. As a result $v_r^{mp}/v_r^N < 1$ and $t^{mp}/t^N > 1$.

Let us examine two suspensions with given parameters $\tilde{k}_1$ and $\tilde{k}_2$. A reduction in $\tilde{k}$ and an increase in $\varkappa$ correspond to an increase in the intrinsic rotation of the fluid's microelements. The larger $\varkappa$, the easier it is for the particles "to escape" over the boundary surface, i.e., the closer the permissible value of α is to 1. On the other hand, if for

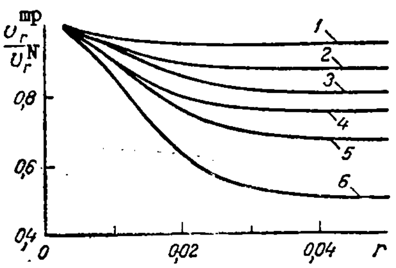

Fig. 1

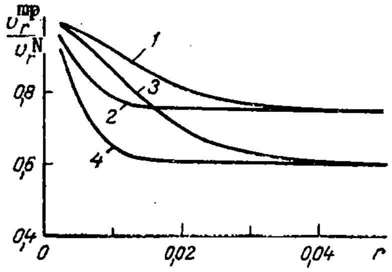

Fig. 2

Fig. 1. The ratio v_r^{mp}/v_r^N as a function of the radius r of the wetted surface circle at $\bar{k} = 5\cdot10^4$ m^{-1}: 1, 2, 4) $\alpha = 0.4$; 3, 5, 6) 0; 1, 3) $\varkappa/\mu^N = 0.5$; 2, 5) 1; 4, 6) 2. r, m.

Fig. 2. The ratio v_r^{mp}/v_r^N as a function of the radius r of the spreading circle for suspensions with $\bar{k} = 5\cdot10^4$ m^{-1} (1, 3) and $\bar{k} = 7\cdot10^3$ m^{-1} (2, 4) when $\alpha = 0.2$, $\varkappa/\mu^N = 1$ (1, 2), 2 (3, 4).

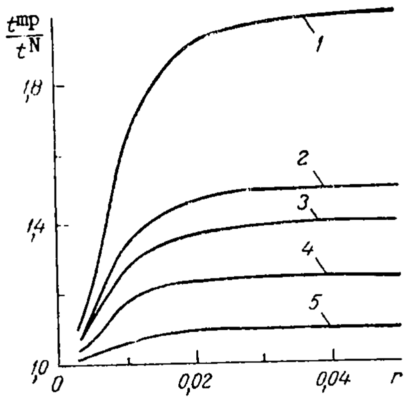

Fig. 3

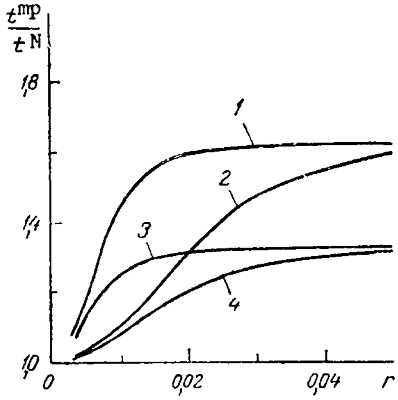

Fig. 4

Fig. 3. The ratio t^{mp}/t^N as a function of r for $\bar{k} = 7\cdot10^3$ m^{-1} and $\alpha = 0$ for $\varkappa/\mu^N = 2$ (1), 1 (2), 0.8 (3), 0.5 (4), 0.2 (5).

Fig. 4. The ratio t^{mp}/t^N as a function of the radius r of the spreading circle for two suspensions at $\varkappa/\mu^N = 2$ and for various boundary conditions: $\alpha = 0.2$ (1, 2), 0.4 (3, 4); $\bar{k} = 7\cdot10^3$ m^{-1} (1, 3), $5\cdot10^4$ m^{-1} (2, 4).

a known $\bar{k}$ we specify α close to 1, it follows from the condition $v_r^{mp}/v_r^N < 1$ and (21) that such a situation is attainable only when $\varkappa > 2\mu(2\alpha - 1)/(1 - \alpha)$.

Following similar considerations, we can demonstrate that if the quantities $\bar{k}$ and $\varkappa$ are given, the boundary-condition parameter α must fall within the range $0 < \alpha < (2\mu^N + \varkappa)/(4\mu^N + \varkappa)$.

Results of a numerical analysis* of formulas (10)-(13), characterizing the effect of fluid micropolarity on the kinetics of fluid spreading, are shown in Figs. 1-4.

As we can see from Fig. 1, the closer the boundary conditions are to the case of total adhesion, when $v_r = v_\varphi = 0$, and also the larger the values of $\varkappa$, the greater the difference between v_r^{mp} and v_r^N.

Analysis of Fig. 2 shows that, beginning from some value for the spreading radius, for various values of $\bar{k}$ and the same values of $\varkappa$ and α, the quantity v_r^{mp}/v_r^N becomes constant. This is associated with the fact that the effect of microrotations is determined not only by the quantity $\bar{k}$, but by the ratio $\xi = p/r^2$, which can be presented in the form $\xi = kh$.

*The authors express their gratitude to Dr. B. Altan for the numerical calculations.

Therefore, suspensions with various values of $\tilde{k}_1$ and $\tilde{k}_2$ for corresponding spreading radii r_1 and r_2 have identical ξ, in which case for small values of ξ the relationship between v_r^{mp}/v_r^N and that quantity disappears, as follows from (21). It is obvious that the maximum micropolarity of the fluid (Fig. 2) makes its appearance for a smaller value of $\tilde{k}$, the smaller value of the radius of the spreading spot.

Figure 3 shows some results from an analysis carried out with the aid of formula (13) into the effect of fluid micropolarity on the time it takes for the drop spreading front to reach corresponding values for the wetted-spot radius. The curves have been plotted for the case of extremely pronounced influence from the boundary surface on the intrinsic rotations of the fluid particles ($\alpha = 0$) for various values of $\varkappa$. We see, for example, that when $\varkappa = 2\mu^N$ and $\tilde{k} = 7 \cdot 10^3$ m^{-1}, in this case even with a spot radius of $r = 10^{-2}$ m the difference between t^{mp} and t^N reaches more than 60%.

Figure 4 illustrates the kinetics of spreading for the suspensions that we are considering here, for various values of the boundary-condition parameter. The difference between t^{mp} and t^N increases with a decrease in α and $\tilde{k}$, and here, beginning with some value of r, it is independent of the radius for a variety of fluids.

The results that we have obtained demonstrate that with comparable dimensions for the spreading-layer thickness and the particle diameters the intrinsic rotations of the latter may exert significant effect on the kinetics of drop spreading over a horizontal solid surface. The proposed method of measuring the characteristics of fluid micropolarity makes it possible quantitatively to evaluate this influence.

NOTATION

ρ, the fluid density; t, time; λ, μ, $\varkappa$, γ, α_V, and β_V, material constants of the micropolar fluid; $\mathbf{f}$, mass force; $\mathbf{l}$, body couple vector; j, microinertia; π^0, thermodynamic pressure; v_r, ν_φ, nonzero components of the velocity and microrotation vectors; m, drop mass; r, radius of the circle formed on the wetted surface; α, boundary-condition parameter for the microrotation vector; $\mathbf{t}$ and $\mathbf{m}$, stress and couple-stress tensors; h, thickness of the spreading layer; d, dimension of the fluid microelement.

LITERATURE CITED

1. E. B. Dussan and S. H. Davis, J. Fluid Mech., 61, No. 1, 71-95 (1974).
2. E. B. Dussan, Ann. Rev. Fluid Mech., 11, 371-400 (1979).
3. L. M. Hocking, Quart. J. Mech. Appl. Math., 34, No. 1, 37-55 (1981).
4. L. M. Hocking and A. D. Rivers, J. Fluid Mech., 121, 425-442 (1982).
5. R. G. Cox, J. Fluid Mech., 168, 169-194 (1986).
6. R. G. Cox, J. Fluid Mech., 168, 195-200 (1986).
7. E. D. Shchukin, Yu. V. Goryunov, G. I. Den'shchikova, et al., Kolloid. Zh., 25, No. 1, 108-114 (1963).
8. É. A. Raud, B. D. Summ, and E. D. Shchukin, Dokl. Akad. Nauk SSSR, 205, No. 5, 1135-1137 (1972).
9. Z. P. Shul'man and B. M. Khusid, Nonsteady Processes of Convective Transfer in Hereditary Media [in Russian], Minsk (1983).
10. A. Adamson, The Physical Chemistry of Surfaces [Russian translation], Moscow (1979).
11. A. C. Eringen, J. Math. Mech., 16, No. 1, 1-18 (1966).
12. B. V. Deryagin, Dokl. Akad. Nauk SSSR, 51, No. 5, 357-360 (1946).
13. N. P. Nigun, Rheologica Acta, 23, 575-581 (1984).
14. N. P. Migun and P. P. Prokhorenko, The Hydrodynamics and Exchange of Heat of the Gradient Flows of a Microstructural Fluid [in Russian], Minsk (1984).